

Phenomenology of neutral D –meson decays and double-flavour oscillations

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Abstract. Decays of neutral D -mesons are considered phenomenologically without invoking any particular models. Special attention is given to cascade decays with intermediate neutral kaons where coherent double-flavour oscillations (CDFO) become possible. We show necessity and unique possibilities of experiments on CDFO. They allow to relate with each other widths and masses of D -meson eigenstates, to separate interference effects due to D^0 - \bar{D}^0 mixing and/or Cabibbo-favoured vs. doubly-suppressed transitions. Such experiments provide the only known ways to unambiguous model-independent measurements of all CP -violating parameters and of Cabibbo-doubly-suppressed amplitudes, where the New Physics may have more prominent manifestations. Similar experiments would be useful and interesting also for charged D -meson decays to neutral kaons.

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1 Introduction

Study of coherent double-flavour oscillations (CDFO) was suggested some years ago[1] as a method for detailed investigations of properties of heavy mesons. The phenomenon emerges if a secondary neutral kaon produced in decay of a heavier neutral flavoured meson evolves so as to coherently continue the pre-decay evolution of the initial heavy meson. It has been discussed in a number of papers [1-7], mainly for B -mesons. The method suggests new tools to measure Δm and $\Delta\Gamma$ for B_d [1,2] and B_s [5] mesons, providing, in particular, a unique possibility to find their signs. For experimental studies of CP -violation it can present a practical way to measure CP -violating parameters in neutral B -meson decays unambiguously and independently of any model-based assumptions[3,6,7]. Detailed discussion of these and other aspects of CDFO in B -decays may be found in the review talk[8]. The problem of ambiguities for parameters of CP -violation has recently been discussed also in a large number of papers (see, e.g., [9-14]).

The present paper concerns with special features of D -meson decays which appear to be, in some sense, phenomenologically more general than B -meson ones. D -meson physics has many interesting problems (see, e.g., the detailed mini-review[15]). One of essential phenomenological differences between B - and D -physics is that any particular decay of B -mesons corresponds to a

single-flavour transition, while D -mesons may have various flavour transitions in the same decay mode. As noted in[16,17] (see also[18]), specifics of neutral kaons as decay products may generate unusual effects even in decays of charged (and, surely, unmixed) D^\pm -mesons. Their source is that in such decays the Cabibbo-allowed and doubly-suppressed transitions, which are just different flavour transitions, become coherent. As a result, in particular, the sought-for D -meson CP -violation effects become observationally mixed with the well-studied kaon ones.

In neutral D -decays the mixing of D^0 and \bar{D}^0 opens possibility of CDFO and leads to additional non-standard effects. So, in analogy with B -meson decays considered earlier[1], we are interested now in cascade decays of the type

$$D^0(\bar{D}^0) \rightarrow XK^0(\bar{K}^0), \quad (1)$$

with subsequent kaon decays; X is a neutral system with definite values of spin and CP -parity. Our aim is to study what physical information may be extracted from double-time distributions over primary and secondary lifetimes t_D and t_K .

Decays (1) are mainly induced by the quark transitions $c \rightarrow sW^+$, $\bar{c} \rightarrow \bar{s}W^-$, which produce meson transitions

$$D^0 \rightarrow \bar{K}^0, \quad \bar{D}^0 \rightarrow K^0. \quad (2)$$

Their final strangeness is the same as in decays of \bar{B}_d , B_d or B_s , \bar{B}_s , studied in papers[1,5] respectively. Hence, if

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only transitions (2) existed we could apply ready expressions from those papers to describe time distributions of decays (1). However, transitions

$$D^0 \rightarrow K^0, \quad \overline{D}^0 \rightarrow \overline{K}^0, \quad (3)$$

with the “wrong” final strangeness, are also possible. Being induced by the quark cascades $c \rightarrow dW^+$, $W^+ \rightarrow u\bar{s}$ and charge conjugate, they are doubly Cabibbo-suppressed. Nevertheless, when searching for very small expected effects of D -meson mixing or CP -violation, the interference of transitions (2) and (3) in the secondary kaon decays should be taken into account. Moreover, the doubly-suppressed transitions are a new kind of manifestations of electro-weak interactions, which may reveal some New Physics; so their studies are of independent interest. Therefore, we begin here with exact expressions and apply smallness assumptions only later. (Note that in particular decays of B -mesons the “wrong”-strangeness transitions are practically absent being suppressed much stronger.) At first sight, possibility of two transitions (2) and (3) might essentially complicate time distributions, in comparison with B -mesons. We will see, however, that the complications are, in essence, not very serious. Moreover, they open new possibilities to extract physically interesting information from experiments.

The further presentation goes as follows. In Section 2 we give general description of cascades initiated by decays (1) with subsequent kaon decays. Physical content of seemingly complicated expressions is first discussed in Section 3 for the simplified case of exact CP -conservation. For the realistic case of violated CP -parity we explain in Section 4 that physical identification of neutral D -meson eigenstates is important to prevent ambiguities both in measuring CP -violation parameters and in separating amplitudes of flavour transitions (2) and (3). For illustration we consider two kinds of labeling the eigenstates. In Section 4 they are marked as being approximately CP -even or CP -odd, while in Section 5 we label them by the heavier or lighter mass. We show that in both cases the double-time decay distributions of cascades (1) are necessary and sufficient to relate together various properties of eigenstates and eliminate ambiguities from measurements of physical quantities. To conclude we summarize the results and briefly discuss possible strategies of experiments.

2 General formalism

Neutral D -mesons produce two eigenstates which we denote by D_{\pm} (the meaning of such notations is discussed below). For simplicity we assume CPT (but not CP) invariance. Then the eigenstates may be written as

$$D_{\pm} = p_D D^0 \pm q_D \overline{D}^0; \quad (4)$$

they have definite masses and widths; simple factors

$$e_{\pm}(t) = \exp(-im_{\pm}t - \Gamma_{\pm}t/2)$$

describe their time evolutions. The relations (4) are considered in many papers as definitions of the eigenstates (for kaons, in a standard way, K_S is assumed to be K_+ , while K_L is identified with K_-). We emphasize, however, that these definitions are only formal and cannot be considered as physical definitions of eigenstates. Since coefficients p, q are not assumed to be real, one may actually redefine phases of states so, that any prescribed eigenstate would look as D_+ (for kaons one may consider K_S and K_L as having the form of K_- and K_+ respectively, by changing phases of K^0 and \overline{K}^0 without changing any physical quantities). The problem of true physical definitions for the eigenstates will be considered below.

In analogy with [1], we start, say, with the pure D^0 -state. During the time interval t_D it evolves into the state

$$D(t_D) = \frac{1}{2p_D} [e_+(t_D)D_+ + e_-(t_D)D_-]. \quad (5)$$

Decay (1) at the moment t_D generates the kaon state (up to normalization)

$$K(t_D; 0) = \frac{1}{2p_D} \{ [a_{+S}^{(X)} e_+(t_D) + a_{-S}^{(X)} e_-(t_D)] K_S + [a_{+L}^{(X)} e_+(t_D) + a_{-L}^{(X)} e_-(t_D)] K_L \}, \quad (6)$$

where $a_{\pm S}^{(X)}, a_{\pm L}^{(X)}$ are amplitudes of decays (1) with transitions $\overline{D}_{\pm} \rightarrow K_{S,L}$. Evolution during the time t_K transforms it into

$$K(t_D; t_K) = \frac{1}{2p_D} \{ [a_{+S}^{(X)} e_+(t_D) + a_{-S}^{(X)} e_-(t_D)] e_S(t_K) K_S + [a_{+L}^{(X)} e_+(t_D) + a_{-L}^{(X)} e_-(t_D)] e_L(t_K) K_L \}, \quad (7)$$

with $e_{S,L}(t) = \exp(-im_{S,L}t - \Gamma_{S,L}t/2)$.

Let b_{Sf} and b_{Lf} denote amplitudes of decays

$$K_{S,L} \rightarrow f. \quad (8)$$

Then the cascade, initiated by the pure D^0 -meson and consisting of the primary decay (1) after lifetime t_D and the secondary kaon decay (8) after lifetime t_K , has the probability amplitude equal to

$$A_{D \rightarrow Xf}(t_D; t_K) = \frac{1}{2p_D} \{ [a_{+S}^{(X)} e_+(t_D) + a_{-S}^{(X)} e_-(t_D)] b_{Sf} e_S(t_K) + [a_{+L}^{(X)} e_+(t_D) + a_{-L}^{(X)} e_-(t_D)] b_{Lf} e_L(t_K) \}. \quad (9)$$

The amplitude for the similar cascade initiated by the pure \overline{D}^0 -meson is somewhat different. It equals

$$A_{\overline{D} \rightarrow Xf}(t_D; t_K) = \frac{1}{2q_D} \{ [a_{+S}^{(X)} e_+(t_D) - a_{-S}^{(X)} e_-(t_D)] b_{Sf} e_S(t_K) + [a_{+L}^{(X)} e_+(t_D) - a_{-L}^{(X)} e_-(t_D)] b_{Lf} e_L(t_K) \}. \quad (10)$$

Double-time distributions of these cascades may be presented in the form

$$\begin{aligned} W^{Xf}(t_D; t_K) &= |A_{D \rightarrow Xf}(t_D; t_K)|^2; \\ \overline{W}^{Xf}(t_D; t_K) &= |A_{\overline{D} \rightarrow Xf}(t_D; t_K)|^2. \end{aligned} \quad (11)$$

Their structure reminds that described in [1]. The distributions are not factorisable. As a function of one time (say, of t_D) they are linear combinations of four terms

$$\begin{aligned} &\exp(-\Gamma_+ t_D), \exp[-(\Gamma_+ + \Gamma_-)t_D/2] \cos(\Delta m_D t_D), \\ &\exp(-\Gamma_- t_D), \exp[-(\Gamma_+ + \Gamma_-)t_D/2] \sin(\Delta m_D t_D). \end{aligned}$$

Coefficients depend on t_K and, in their turn, are also linear combinations of similar terms

$$\begin{aligned} &\exp(-\Gamma_S t_K), \exp[-(\Gamma_S + \Gamma_L)t_K/2] \cos(\Delta m_K t_K), \\ &\exp(-\Gamma_L t_K), \exp[-(\Gamma_S + \Gamma_L)t_K/2] \sin(\Delta m_K t_K). \end{aligned}$$

Physically more transparent is a different way of describing the double-time distributions (11). They contain several various contributions. First of all, there are non-interfering contributions of 4 possible cascade branches (corresponding to various combinations of subscripts in $D_{\pm} \rightarrow K_{S,L}$):

$$\begin{aligned} |2p_D|^2 W_{noint}^{Xf}(t_D; t_K) &= |2q_D|^2 \overline{W}_{noint}^{Xf}(t_D; t_K) \\ &= |a_{+S}^{(X)} b_{Sf}|^2 \exp(-\Gamma_+ t_D - \Gamma_S t_K) \\ &\quad + |a_{+L}^{(X)} b_{Lf}|^2 \exp(-\Gamma_+ t_D - \Gamma_L t_K) \\ &\quad + |a_{-S}^{(X)} b_{Sf}|^2 \exp(-\Gamma_- t_D - \Gamma_S t_K) \\ &\quad + |a_{-L}^{(X)} b_{Lf}|^2 \exp(-\Gamma_- t_D - \Gamma_L t_K). \end{aligned} \quad (12)$$

Then, there are single-interference contributions. They are due to $K_{S,L}$ interference without D_{\pm} interference or, vice versa, due to interference of D_{\pm} without $K_{S,L}$ interference:

$$\begin{aligned} |2p_D|^2 W_{Kint}^{Xf}(t_D; t_K) &= |2q_D|^2 \overline{W}_{Kint}^{Xf}(t_D; t_K) \\ &= 2\text{Re}[a_{+L}^{(X)*} a_{+S}^{(X)} b_{Lf}^* b_{Sf} \exp(i\Delta m_K t_K)] \\ &\quad \times \exp[-\Gamma_+ t_D - (\Gamma_S + \Gamma_L)t_K/2] \\ &\quad + 2\text{Re}[a_{-L}^{(X)*} a_{-S}^{(X)} b_{Lf}^* b_{Sf} \exp(i\Delta m_K t_K)] \\ &\quad \times \exp[-\Gamma_- t_D - (\Gamma_S + \Gamma_L)t_K/2]; \end{aligned} \quad (13)$$

$$\begin{aligned} |2p_D|^2 W_{Dint}^{Xf}(t_D; t_K) &= -|2q_D|^2 \overline{W}_{Dint}^{Xf}(t_D; t_K) \\ &= 2|b_{Sf}|^2 \text{Re}[a_{-S}^{(X)*} a_{+S}^{(X)} \exp(i\Delta m_D t_D)] \\ &\quad \times \exp[-(\Gamma_+ + \Gamma_-)t_D/2 - \Gamma_S t_K] \\ &\quad + 2|b_{Lf}|^2 \text{Re}[a_{-L}^{(X)*} a_{+L}^{(X)} \exp(i\Delta m_D t_D)] \\ &\quad \times \exp[-(\Gamma_+ + \Gamma_-)t_D/2 - \Gamma_L t_K]. \end{aligned} \quad (14)$$

And, at last, there are double-interference contributions, which contain interference of both D_{\pm} and $K_{S,L}$:

$$\begin{aligned} |2p_D|^2 W_{DKint}^{Xf}(t_D; t_K) &= -|2q_D|^2 \overline{W}_{DKint}^{Xf}(t_D; t_K) \\ &= 2\{\text{Re}[a_{-L}^{(X)*} a_{+S}^{(X)} b_{Lf}^* b_{Sf} \exp(i\Delta m_D t_D + i\Delta m_K t_K)] \\ &\quad + \text{Re}[a_{+L}^{(X)*} a_{-S}^{(X)} b_{Lf}^* b_{Sf} \exp(-i\Delta m_D t_D + i\Delta m_K t_K)]\} \\ &\quad \times \exp[-(\Gamma_+ + \Gamma_-)t_D/2 - (\Gamma_S + \Gamma_L)t_K/2]. \end{aligned} \quad (15)$$

In (13)–(15) we have used

$$\Delta m_K = m_L - m_S, \Delta m_D = m_- - m_+. \quad (16)$$

Of course, we consider K -meson decay amplitudes b_{Sf} and b_{Lf} as known from previous experiments.

Now, if $\Gamma_+ \neq \Gamma_-$ and $\Delta m_D \neq 0$, eqs.(12)–(15) show that the double-time distributions for the cascade (1), (8) contain 10 terms with different time-dependence, which can be separated from each other. They allow to measure 4 absolute values of decay amplitudes $a_{\pm S}^{(X)}, a_{\pm L}^{(X)}$ and 6 their relative phases. To end this section, we express those eigenstate amplitudes in terms of flavour amplitudes corresponding to flavour transitions (2), (3):

$$2a_{+S}^{(X)} = \frac{p_D}{p_K} a_{DK}^{(X)} + \frac{q_D}{p_K} a_{\overline{DK}}^{(X)} + \frac{p_D}{q_K} a_{D\overline{K}}^{(X)} + \frac{q_D}{q_K} a_{\overline{D}\overline{K}}^{(X)}; \quad (17)$$

$$2a_{-L}^{(X)} = \frac{p_D}{p_K} a_{DK}^{(X)} - \frac{q_D}{p_K} a_{\overline{DK}}^{(X)} - \frac{p_D}{q_K} a_{D\overline{K}}^{(X)} + \frac{q_D}{q_K} a_{\overline{D}\overline{K}}^{(X)}; \quad (18)$$

$$2a_{+L}^{(X)} = \frac{p_D}{p_K} a_{DK}^{(X)} + \frac{q_D}{p_K} a_{\overline{DK}}^{(X)} - \frac{p_D}{q_K} a_{D\overline{K}}^{(X)} - \frac{q_D}{q_K} a_{\overline{D}\overline{K}}^{(X)}; \quad (19)$$

$$2a_{-S}^{(X)} = \frac{p_D}{p_K} a_{DK}^{(X)} - \frac{q_D}{p_K} a_{\overline{DK}}^{(X)} + \frac{p_D}{q_K} a_{D\overline{K}}^{(X)} - \frac{q_D}{q_K} a_{\overline{D}\overline{K}}^{(X)}. \quad (20)$$

Also useful may be other combinations of amplitudes. For instance, transitions $D \rightarrow K_{S,L}$ and $\overline{D} \rightarrow K_{S,L}$ can be described by the amplitudes

$$a_{DS} = \frac{a_{DK}}{2p_K} + \frac{a_{D\overline{K}}}{2q_K} = \frac{a_{+S} + a_{-S}}{2p_D}, \quad (21)$$

$$a_{DL} = \frac{a_{DK}}{2p_K} - \frac{a_{D\overline{K}}}{2q_K} = \frac{a_{+L} + a_{-L}}{2p_D};$$

$$a_{\overline{DS}} = \frac{a_{\overline{DK}}}{2p_K} + \frac{a_{\overline{D}\overline{K}}}{2q_K} = \frac{a_{+S} - a_{-S}}{2q_D}, \quad (22)$$

$$a_{\overline{DL}} = \frac{a_{\overline{DK}}}{2p_K} - \frac{a_{\overline{D}\overline{K}}}{2q_K} = \frac{a_{+L} - a_{-L}}{2q_D}.$$

Single-transition cases simplify the eigenstate amplitudes. For the pure transition (2)

$$\begin{aligned} a_{+S}^{(X)} &= -a_{-L}^{(X)}, a_{-S}^{(X)} = -a_{+L}^{(X)}; \\ a_{DS} &= -a_{DL}, a_{\overline{DS}} = a_{\overline{DL}}; \end{aligned} \quad (23)$$

while for the pure transition (3)

$$\begin{aligned} a_{+S}^{(X)} &= a_{-L}^{(X)}, a_{-S}^{(X)} = a_{+L}^{(X)}; \\ a_{DS} &= a_{DL}, a_{\overline{DS}} = -a_{\overline{DL}}. \end{aligned} \quad (24)$$

In the general case of two flavour transitions with CP -violation, all four eigenstate (or flavour) amplitudes become independent.

3 The case of CP -conservation

Now we discuss physical meaning of the obtained expressions in more detail. At first, for simplicity, we begin with exact conservation of CP -parity which will be assumed throughout the present section, both for D -mesons and for kaons. Then, the eigenstates $K_{S,L}$ have definite CP -parities equal to ± 1 respectively. D -meson eigenstates should also have definite CP -parities, and we suggest in this section that the indices of D_{\pm} just label CP -parities ± 1 of the eigenstates.

In what follows we need to fix final states for decays (1) and (8). As the first stage of the cascades we can use decays

$$D^0(\bar{D}^0) \rightarrow (\pi^0, \eta, \eta', \rho^0, \omega, \phi) + K^0(\bar{K}^0). \quad (25)$$

These final states may be produced by various decay mechanisms but look similar in terms of the formalism of the preceding section. Since we assume the exact CP -conservation, all the final states (1) with kaons being in one of their eigenstates have definite CP -parities equal to $(CP)_X(CP)_K(-1)^{S_X}$, where $(CP)_X$ and S_X are CP -parity and spin of the system X . For decays (25) this CP -parity is just opposite to the CP -parity of the corresponding kaon eigenstate $(CP)_K$. So, the above choice of eigenstates D_{\pm} leads, for all X in (25), to vanishing of two amplitudes:

$$a_{+S}^{(X)} = a_{-L}^{(X)} = 0 \quad (26)$$

(this can be seen also from eqs.(17)-(20)). As a result, all single-interference contributions (13), (14) disappear. One of double-interference contributions of (15) disappears as well, but another survives.

This situation corresponds to existence of two independent decay branches (instead of four in a general CP -violating case)

$$D_+ \rightarrow XK_L, D_- \rightarrow XK_S; K_{L,S} \rightarrow f,$$

which can interfere only after the last decay (compare to the similar consideration in [1]). The double-time distributions, $W^{Xf}(t_D; t_K)$ and $\bar{W}^{Xf}(t_D; t_K)$ as defined in (11), consist each of two parts, either without interference of branches or with both D and K interference:

$$\begin{aligned} 4W_{noint}^{Xf}(t_D; t_K) &= 4\bar{W}_{noint}^{Xf}(t_D; t_K) \\ &= |a_{+L}^{(X)} b_{Lf}|^2 \exp(-\Gamma_+ t_D - \Gamma_L t_K) \\ &\quad + |a_{-S}^{(X)} b_{Sf}|^2 \exp(-\Gamma_- t_D - \Gamma_S t_K), \end{aligned} \quad (27)$$

$$\begin{aligned} 4W_{DKint}^{Xf}(t_D; t_K) &= -4\bar{W}_{DKint}^{Xf}(t_D; t_K) \\ &= 2\text{Re}[a_{+L}^{(X)*} a_{-S}^{(X)} b_{Lf}^* b_{Sf}] \\ &\quad \times \exp(-i\Delta m_D t_D + i\Delta m_K t_K)] \\ &\quad \times \exp[-(\Gamma_+ + \Gamma_-)t_D/2 \\ &\quad - (\Gamma_S + \Gamma_L)t_K/2]. \end{aligned} \quad (28)$$

One part is monotone (independent contributions of the decay branches), another oscillates (interference of the branches). These parts can be easily separated by considering sum or difference of W^{Xf} and \bar{W}^{Xf} .

The monotone terms determine D -meson eigenwidths and absolute values of amplitudes $|a_{+L}^{(X)}|$, $|a_{-S}^{(X)}|$. Note that CP -conservation makes the D -meson indices of amplitudes and lifetimes be directly and unambiguously related to the corresponding kaon indices. This means that we can easily determine CP -parity for any D -eigenstate. Of course, this CP -parity is just the final state CP -parity (we emphasize that it is single-valued for decays (25) with kaon in an eigenstate, when CP -conservation is exact). The situation is the same as in ascribing CP -parities to K_S and K_L through their decays to 2π or 3π . Moreover, this strict correlation between kaon and D -meson indices means that the monotone terms directly determine relation between an eigenlifetime and CP -parity of the corresponding eigenstate (this problem would not be so simple in the general case of CP -violation; see following sections for more details).

The oscillating term allows to determine the sign of Δm_D in respect to the known sign of Δm_K . In other words, it determines which of the D -meson eigenstates, CP -even or CP -odd, is heavier or lighter. The coefficient of the oscillating term checks consistency of absolute values of the two non-vanishing amplitudes, while the constant phaseshift of oscillations determines the relative phase of these amplitudes. Note that this double-oscillation (in t_D and t_K) is, in essence, similar to secondary oscillations in kaon regeneration [19] which opened possibility to determine the sign of Δm_K in respect to the sign of the regeneration phase. We emphasize that the oscillating term allows to relate the heavier or lighter mass to eigenstate CP -parities, but not directly to longer or shorter lifetimes.

The above expressions have seemingly the same form as for B -meson decays [1] where the final strangeness is strictly correlated with the initial flavour. The real difference is the independence of amplitudes $a_{-S}^{(X)}$ and $a_{+L}^{(X)}$ if both transitions, (2) and (3), are present. As a result, the two nonvanishing flavour transitions lead to $|a_{+L}^{(X)}| \neq |a_{-S}^{(X)}|$.

There is one more consequence: complexities of the amplitudes $a_{+L}^{(X)}$ and $a_{-S}^{(X)}$ (or, equivalently, of the amplitudes $a_{DK}^{(X)}$ and $a_{D\bar{K}}^{(X)}$) may be, generally, different. This statement looks rather evident for the final states $\pi^0 K^0(\bar{K}^0)$ or $\rho^0 K^0(\bar{K}^0)$ which combine two isotopic-spin states. It is less familiar but also true for such final states as, say, $\omega K^0(\bar{K}^0)$ which are different components of the same isotopic-spin state. The reason is that the standard idea of the decay amplitude having the same phase as the elastic scattering amplitude for hadrons in the decay final state is not always correct. It is true only if the final-state interaction (FSI) cannot rescatter the particular state into some other states. However, the D -meson mass is high enough,

and any particular final state in D -decay does can rescatter (most evidently, $K\omega$ may rescatter into $K3\pi$ with pions out of resonance).

Formally, this means that any particular final state in decays (25) does not diagonalize the strong-interaction S -matrix and does not produce a universal FSI-phase. The amplitudes $a_{DK}^{(X)}$ and $a_{D\bar{K}}^{(X)}$ appear to be some linear combinations of amplitudes for transitions into combined states which diagonalize the strong S -matrix (and produce universal FSI-phases). Since the mechanisms of transitions (2) and (3) are different, the two combinations are also different and, therefore, final-state interactions may generate different phases for the two amplitudes. Thus, the factor $a_{+L}^{(X)*} a_{-S}^{(X)}$ in (28) is real if only one of the transitions, (2) or (3), is operative, but may be, generally, complex if the both transitions are possible. Experimental determination of such phase difference could be useful to reveal possible decay mechanisms.

Let us discuss specific final states for the second stage (8) of cascades. If we take $f = 2\pi$, then $b_{L(2\pi)} = 0$ and the double-time distributions (11) contain only one term, expressed through $|a_{-S}^{(X)}|^2$. For $f = 3\pi$ we have $b_{S(3\pi)} = 0$. The distributions, again, contain only one term, expressed through $|a_{+L}^{(X)}|^2$. These two final states lead just to the situations discussed in [17]. Cascades with the two final states allow one to measure absolute values of the corresponding first-stage amplitudes, but not their relative phase. Therefore, they do not allow to find amplitudes $a_{DK}^{(X)}$ and $a_{D\bar{K}}^{(X)}$ unambiguously.

Semileptonic kaon decays with $f = \pi^\mp l^\pm \nu(\bar{\nu})$ have $|b_{Lf}| = |b_{Sf}|$, and all three terms of eqs.(27), (28) appear in the double-time distributions. Generally, they have different dependence on t_D which might help to separate them. In any case, the three terms have different dependence on t_K and may be separated for sure. Then, one can determine here not only absolute values of $a_{+L}^{(X)}$ and $a_{-S}^{(X)}$, but their relative phase as well. In other words, cascades with semileptonic secondary decays allow one to unambiguously find both the absolute values and relative phase of the amplitudes $a_{DK}^{(X)}$ and $a_{D\bar{K}}^{(X)}$ for the primary decays (25).

Thus, investigation of double-time distributions in cascade decays (25), (8) may solve several important problems: it measures the D -meson eigenwidths Γ_\pm and mass difference Δm_D , relates eigenwidths and eigenmasses to each other and to CP -parities of eigenstates, determines amplitudes $a_{D\bar{K}}^{(X)}$ and $a_{DK}^{(X)}$ (together with their relative phase) for the favoured and suppressed flavour transitions (2), (3).

Earlier, in [1,3], we noticed that interesting problems for B -mesons may be attacked also in single-time distributions over t_K (integrated over t_B). Presence of two transitions in D -meson decays produces more of independent amplitudes and makes single-time distributions less efficient. Consider, e.g., contribution (28). After integration

over t_D it contains the factor

$$\cos(\phi_{SL}^{(Xf)} - \alpha_D + \Delta m_K t_K), \quad (29)$$

where $\phi_{SL}^{(Xf)}$ is the relative phase of $a_{-S}^{(X)} b_{Sf}$ and $a_{+L}^{(X)} b_{Lf}$, while

$$\tan(\alpha_D/2) = x_D \equiv \frac{2\Delta m_D}{\Gamma_+ + \Gamma_-}.$$

Single-time distributions in D -meson decays provide no way to separate $\phi_{SL}^{(Xf)}$ from α_D . Note, for comparison, that the B -meson analog of $\phi_{SL}^{(Xf)}$ has a definite value depending on spin and CP -parity of the system X and on the final state f in the secondary kaon decay (it is 0 or π for semileptonic kaon decays). This is the reason why α_D cannot be measured model-independently in single-time decay distributions for neutral D -mesons, while similar distributions in decays of neutral B -mesons may be sufficient to measure an analogous quantity α_B [3].

If Δm_D and $\Delta\Gamma = \Gamma_+ - \Gamma_-$ are vanishing (or too small to be measured) then the three terms in contributions (27), (28) have the same t_D -dependence. Neutral D -mesons in this situation are unmixed, and so, their decays exactly correspond to such decays of charged D -mesons as, e.g.,

$$D^\pm \rightarrow (\pi^\pm, \rho^\pm) + K^0(\bar{K}^0) \quad (30)$$

with subsequent semileptonic kaon decays. The three terms in time distributions can still be separated by their different t_K -dependence. Note that the single-time distribution in t_K is sufficient here to separate and measure amplitudes of transitions (2), (3) and their relative phase. Thus, for unmixed D -mesons the secondary-decay distribution appears to be even more interesting than the primary-decay one. For the above measurements one does not need to study the large- t_K region ($t_K \gtrsim \tau_L$). Necessary is only the interval of t_K up to about $(10 - 15)\tau_S$, overlapping the $K_{S,L}$ interference region.

4 CP -parity eigenstates with CP -violation

To consider the general case which corresponds to violated CP -parity we return to exact expressions (12)-(15) for double-time distributions. They contain 10 different terms which, in principle, determine absolute values of 4 amplitudes $a_{\pm S}^{(X)}, a_{\pm L}^{(X)}$ and 6 their relative phases. To interpret results of measurements, some physical identification of eigenstates appears to be necessary. For clarification of this point let us compare kaons and heavier mesons.

First of all, note that every meson eigenstate has 3 main characteristics: width, mass and CP -parity (at least, approximate). Thus, we have 3 different ways of labeling two eigenstates by 3 corresponding pairs: shorter or longer lifetime; lighter or heavier mass; even or odd CP -parity. Of course, these ways are physically equivalent, but the equivalence can be realized only if one has experimental methods to relate those 3 characteristics with each other.

The kaon eigenstates, $K_{S,L}$, are usually identified and labeled by their lifetimes, shorter or longer. Their prevailing hadronic decay modes, 2π or 3π , determine their

CP -parity, at least approximately. The mass difference of K_S and K_L can be easily measured in semileptonic decays. On this way, however, one cannot find the sign of Δm_K , i.e., to determine which of the states is heavier or lighter. Only the specially invented (and rather complicated) experiments allowed to measure the sign of mass difference Δm_K and, thus, relate widths and masses of K_S and K_L to each other [19]; more detailed theoretical references and compilation of experimental results see in [20]). This provided possibility for unambiguous measuring the kaon CP -violating parameters (note that the signs of ϕ_{+-} and ϕ_{00} , the phases of η_{+-} and η_{00} , are measured only in respect to the sign of Δm_K). As a result, we have now, indeed, at least three equivalent ways of identifying $K_{S,L}$: by shorter or longer lifetime, by heavier or lighter mass, by the (approximate) CP -parity.

The first of these ways cannot be applied at present to D -meson eigenstates, D_{\pm} , because of very small (and yet unobserved) difference of eigenlifetimes (the same is true for B -mesons). The absolute value of Δm has been measured for B_d -mesons (results and references see in [21]), there are suggestions how to do this for D -mesons as well. In contrast to kaons, for B - and D -mesons the values of $|\Delta m|$ are expected to be noticeably higher than $|\Delta \Gamma|$ (it is definitely so for B -mesons). Therefore, a rather familiar way in the current literature is to identify eigenstates, for both B and D , as heavier and lighter (i.e., $B_{h,l}$ and $D_{h,l}$). On the other side, identification of eigenstates by their approximate CP -parities was suggested in [1] for B -mesons and may be applied to D -mesons as well. The real problem is how to relate with each other the different approaches to the B - and/or D -meson eigenstates. For B -mesons it was discussed in [8]. Here we consider the situation for D -mesons and the role of their cascade decays.

For definiteness, we use at the first stages of cascades the same decays (25) as in the preceding section. At the second stages we may also use, as before, the three typical kinds of kaon decays: either semileptonic, or purely pionic with 2 or 3 pions produced. As we have seen, only semileptonic kaon decays could allow to measure the relative phase of amplitudes for CP conserved. On the contrary, with violated CP we might, principally, use any of the three decay modes, since for all of them $|b_{Sf}|, |b_{Lf}| \neq 0$. However, decays $K^0(\bar{K}^0) \rightarrow 3\pi$ are still really useless because of too small $|b_{S(3\pi)}|$. When comparing decays $K^0(\bar{K}^0) \rightarrow 2\pi$ to semileptonic ones, the semileptonic decays may appear experimentally more favourable, by the same arguments as suggested in B -meson studies [3, 7, 8]. This problem, however, will not be discussed here anymore since it requires detailed investigation for a particular detector.

To discuss possible measuring procedures we begin with a hypothetical suggestion that eigenwidths Γ_+ and Γ_- are different enough, so that every term in expressions (12)-(15) can be extracted and studied separately. We also assume that all K -meson parameters and decay amplitudes are known. Then, first of all, from monotone terms of (12) we find two eigenwidths Γ_{\pm} and four absolute val-

ues of amplitudes $|a_{\pm S}^{(X)}|, |a_{\pm L}^{(X)}|$. Their kaon indices S, L are fixed by the corresponding exponentials in t_K . However, this is not so for D -meson indices \pm , which (contrary to the CP -conservation case) are not unambiguously related with kaon ones and not determined by t_K -dependence.

Now we can specify possible meaning of the indices \pm , which has not been fixed yet, and define how to ascribe them to amplitudes and eigenwidths. If CP -violation is small indeed (or at least effectively), we may fix the indices as showing approximate CP -parities of eigenstates. Namely, in such a case there should be two larger and two smaller amplitudes, and the indices \pm of the decaying eigenstates may be ascribed (for states X in decays (25)) so that larger amplitudes conserve CP -parity:

$$|a_{-S}^{(X)}| > |a_{+S}^{(X)}|, |a_{+L}^{(X)}| > |a_{-L}^{(X)}|. \quad (31)$$

Note that in presence of only one transition, (2) or (3), we have $|a_{\pm S}^{(X)}| = |a_{\mp L}^{(X)}|$ (see eqs.(23),(24)), and only one of the inequalities is independent.

At first sight, the two inequalities (31) look trivial even for a general case, since in every pair of amplitudes one of their absolute values is, as a rule, greater than another. However, an essential and nontrivial property of the inequalities is that the two larger amplitudes must correspond to different eigenstates of both kaons and D -mesons (in the expression (12) their monotone contributions should contain exponentials in t_D and t_K with ‘‘opposite’’ combinations of D -meson and kaon eigenwidths in the exponents; this should and may be checked). Really, one pair of amplitudes with the same kaon index (e.g., S) would be sufficient to ascribe indices \pm to the D -meson states. Then the corresponding t_D -exponentials determine, which of D -meson eigenwidths is Γ_+ and which is Γ_- ; in other words, this procedure relates eigenwidths and approximate CP -parities of the eigenstates. After that the indices for another pair of amplitudes are completely fixed, and the second inequality (31) may appear true or false. In the case of small CP -violation it should be true, of course.

If, however, the inequalities are inconsistent, then the choice (31) is contradictory. In such a case the CP -violation in transitions $D_{\pm} \rightarrow K_{S,L}$ could not be considered as effectively small (similar problems for B -mesons are discussed in [1, 3, 8]). The approximate CP -parities of the eigenstates D_{\pm} would become mode-dependent, i.e. the effective CP -parity for the same eigenstate would be different when determined from transitions to K_S or K_L (or some other final states with definite CP -parities). Similar situation is well known for the space-parity violation in weak interactions (recall, that the kaon parity is mode-dependent: it is different when determined from decays $K \rightarrow 2\pi$ or $K \rightarrow 3\pi$).

Now, let us stick to a definite prescription of CP -parities based on a particular decay mode. As the next step we may use two terms of expression (13) to find unambiguously the phases $\arg(a_{+L}^{(X)*} a_{+S}^{(X)})$ and $\arg(a_{-L}^{(X)*} a_{-S}^{(X)})$. Signs of these phases are determined in respect to the known sign of Δm_K . Note that if only one of transitions, (2) or (3), is operative then the two phases differ only by the sign.

If we use two terms of expression (14) to find $\arg(a_{-S}^{(X)*} a_{+S}^{(X)})$ and $\arg(a_{-L}^{(X)*} a_{+L}^{(X)})$ we discover that their signs could be measured only in respect to the yet unknown sign of Δm_D . If, again, only one of transitions, (2) or (3), worked, the situation would become definite due to equality of ratios

$$\frac{a_{-S}^{(X)*} a_{+S}^{(X)}}{a_{+L}^{(X)*} a_{+S}^{(X)}} = \frac{a_{-L}^{(X)*} a_{+L}^{(X)}}{a_{-L}^{(X)*} a_{-S}^{(X)}}$$

(which would equal to -1 or +1 respectively for transitions (2) or (3); see relations (23) and (24)). As a result, the relative phase of two terms in expression (14) equals to the relative phase of two terms in expression (13). This phase may be measured from t_K -dependence of the contribution (13), the sign of the phase being determined in respect to Δm_K . Then, t_D -dependence of the contribution (14) determines the sign of Δm_D in respect to the sign of the (now known) phase, i.e. really in respect to the sign of Δm_K . Such determination can be achieved also when both transitions are present, but not so easily, since then the expressions (13) and (14) contain different phases. Note, however, that the contributions (13), (14) are suppressed if CP -violation is small in any sense.

Contributions (15) are, even by themselves, sufficient to find the sign of Δm_D directly in respect to the sign of Δm_K . Indeed, inequalities (31) lead to

$$|a_{+L}^{(X)*} a_{-S}^{(X)}| > |a_{-L}^{(X)*} a_{+S}^{(X)}| \quad (32)$$

and allow to discriminate the two terms in (15); after that the double-time dependence directly compares Δm_D to Δm_K and, in particular, determines their relative sign. Phases $\arg(a_{-L}^{(X)*} a_{+S}^{(X)})$ and $\arg(a_{+L}^{(X)*} a_{-S}^{(X)})$ can be also determined here. They check self-consistency of the procedure since, surely, there should be

$$\begin{aligned} \arg(a_{+L}^{(X)*} a_{+S}^{(X)}) - \arg(a_{-S}^{(X)*} a_{+S}^{(X)}) &= \arg(a_{+L}^{(X)*} a_{-S}^{(X)}), \\ \arg(a_{-L}^{(X)*} a_{-S}^{(X)}) + \arg(a_{-S}^{(X)*} a_{+S}^{(X)}) &= \arg(a_{-L}^{(X)*} a_{+S}^{(X)}). \end{aligned} \quad (33)$$

Even if the choice (31) is contradictory, we still may define eigenstate CP -parities (i.e., ascribe the indices \pm) so to provide the inequality (32).

The procedures described remind what was really done in kaon studies. For each eigenstate they allow to relate together various state's properties: shorter or longer lifetime, positive or negative (approximate) CP -parity, and heavier or lighter mass. Of course, their combination could be fixed also by different (though equivalent) procedures. We emphasize, however, that some physical procedures are necessary and inevitable. Only with such procedures one becomes able to measure flavour-transition amplitudes unambiguously. We will see further in this section that the same is true also for CP -violating parameters.

The physical necessity of CP -parity prescriptions for eigenstates may be traced to the following simple reason. Time dependence (single or double) is always related to eigenstates. On the other side, flavour amplitudes (say,

$a_{DS}^{(X)}, a_{\overline{D}S}^{(X)}$) correspond to flavour states D and \overline{D} , which are linear combinations of eigenstates. D is conventionally considered as

$$D \sim D_+ + D_- .$$

This definition of D is insensitive to accurate identification of eigenstates. \overline{D} , contrary, is proportional to the difference of the eigenstates,

$$\overline{D} \sim D_+ - D_- ,$$

and their interchange would change the sign of \overline{D} . To cope with the conventional relation $\overline{D} = CP(D)$ we should apply some procedure to define CP -parities of eigenstates, and then subtract the CP -odd state from the CP -even one. Without any procedure the state \overline{D} , and various related physical quantities as well, can be determined only up to the sign.

Let us discuss now a more realistic situation when $\Gamma_+ = \Gamma_-$ with available precision. In such a case the four amplitudes $a_{\pm S}^{(X)}, a_{\pm L}^{(X)}$ cannot be determined unambiguously since several contributions have the same t_D -dependence (see, e.g. eqs.(12), (13)) and cannot be completely separated. The t_D -dependence becomes the same for every contribution if Δm_D is also too small and physical discrimination of eigenstates D_{\pm} disappears at all. However, t_K -dependencies of different contributions are still different, and partial separation of various contributions is still possible. Indeed, by comparing decays of initially pure states D^0 and \overline{D}^0 one could separate contributions (12), (13) on one side and (14), (15) on the other. Then, by means of different t_K -dependence we could discriminate (13) from (12) and split (12) into two parts. In the same manner (15) would be discriminated from (14), which is also split into two parts. So, after all, we can split decay time-distributions for D^0 and \overline{D}^0 only to 6 different terms (instead of 10 for $\Delta\Gamma_D \neq 0, \Delta m_D \neq 0$). One may be still able to find amplitudes of transitions (2), (3), but only with additional simplifying assumptions (e.g., neglecting CP -violation or describing it by some special models).

To understand the situation we return to the cascade amplitudes of eqs.(9), (10). If $\Delta\Gamma_D = \Delta m_D = 0$, then $e_-(t_D) = e_+(t_D)$; mixing is absent, and the initial D -meson state decays without evolution. Therefore, more adequate are not the eigenstate amplitudes $a_{\pm L}^{(X)}, a_{\pm S}^{(X)}$, but their combinations $a_{DS}^{(X)}$ and $a_{\overline{D}L}^{(X)}$ determined by eqs.(21) (or $a_{\overline{D}S}^{(X)}$ and $a_{DL}^{(X)}$; see eqs.(22)) which correspond to transitions $D^0 \rightarrow K_{S,L}$ and $\overline{D}^0 \rightarrow K_{S,L}$ without mixing of initial D^0 and/or \overline{D}^0 . Amplitudes of every pair are still coherent to each other, but not coherent to amplitudes of another pair. Therefore, instead of 10 physical quantities we have now only 6 measurable quantities, which are two absolute values and one relative phase in each of the amplitude pairs (21), (22). Thus again, the 6 physically meaningful quantities correspond to 6 separable terms in double-time distributions (12)-(15) at $\Delta\Gamma_D = \Delta m_D = 0$ (when returning to the case of CP -conservation, we would have additional relations $a_{DS}^{(X)} = -a_{\overline{D}S}^{(X)}, a_{DL}^{(X)} = a_{\overline{D}L}^{(X)}$ for

any X in decays (25), further diminishing the number of independent physical quantities).

Note one more specific feature of cascade decays (1), (8). Each of them contains two CP -violating parameters which may be phenomenologically independent. Physically, they correspond to CP -violation in transitions (2) and (3). Even rough estimates[16] show that CP -violation in the suppressed transition (3) may appear greater than in the favoured transition (2). It may also be very sensitive to some New Physics.

As phenomenological CP -violating parameters for cascades (25), (8) one can use, e.g., the ratios

$$\eta_{DS}^{(X)} = \frac{a_{+S}^{(X)}}{a_{-S}^{(X)}}, \eta_{DL}^{(X)} = \frac{a_{-L}^{(X)}}{a_{+L}^{(X)}} \quad (34)$$

vanishing in the limit of exact CP -conservation (for both D -meson and kaon decays). They have the same structure as the standard parameters η_{00} and η_{+-} for neutral kaon decays: each of them is the ratio of two amplitudes, CP -suppressed and CP -favoured, with the same final state and different decaying eigenstates. If CP -parities of D -meson eigenstates can be chosen so to satisfy inequalities (31) then, for the final states (25), both parameters (34) have absolute values smaller than unity. Inconsistency of the inequalities (31) would imply that the absolute value is less than unity for one of the parameters, but greater than unity for another. In any case, CP -parities of eigenstates can be chosen so to satisfy the condition (32). In terms of the η -parameters this condition takes the simple form

$$|\eta_{DS}^{(X)} \eta_{DL}^{(X)}| < 1.$$

Though this inequality looks quite natural, we emphasize that generally it may be nontrivial. Its correctness for a particular system X in decay (1) may always be achieved by a special choice of CP -properties of D -meson eigenstates in this particular decay. Note, however, that such special choice might depend on the system X if inequalities (31) are inconsistent (recall, that inconsistency of (31) would imply mode-dependence of CP -properties for D -meson eigenstates).

One more possible way to describe CP -violation, appropriate for any cascade (1), is to use parameters of the kind

$$\lambda_{DS}^{(X)} = \frac{q_D}{p_D} \frac{a_{\overline{DS}}^{(X)}}{a_{DS}^{(X)}}, \lambda_{DL}^{(X)} = \frac{q_D}{p_D} \frac{a_{\overline{DL}}^{(X)}}{a_{DL}^{(X)}}. \quad (35)$$

CP -violation may be measured by their deviation from CP -conserving values, which are $+1$ or -1 . For the final states (25) the CP -conserving values are clearly seen from the relations

$$\lambda_{DS}^{(X)} = \frac{\eta_{DS}^{(X)} - 1}{\eta_{DS}^{(X)} + 1}, \lambda_{DL}^{(X)} = \frac{1 - \eta_{DL}^{(X)}}{1 + \eta_{DL}^{(X)}}.$$

By using relations (17)-(22) one can easily express the parameters (35) through flavour-transition amplitudes. In the case of a single-flavour transition, (2) or (3), the two

parameters $\lambda_{DS}^{(X)}$ and $\lambda_{DL}^{(X)}$ differ only in sign; they are proportional to the ratio of the corresponding flavour-transition amplitudes for \overline{D} and D . In the presence of both transitions (2) and (3) we still can describe their CP -properties by separate parameters

$$\lambda_{D\overline{K}}^{(X)} = \frac{q_D}{p_D} \frac{q_K}{p_K} \frac{a_{\overline{DK}}}{a_{DK}}, \lambda_{DK}^{(X)} = \frac{q_D}{p_D} \frac{p_K}{q_K} \frac{a_{\overline{DK}}}{a_{DK}}$$

for each transition (just such parameters were used in our papers on B -mesons[1,3,5,6,8]). If CP -violation in the two transitions is the same (i.e. $\lambda_{D\overline{K}}^{(X)} = \lambda_{DK}^{(X)}$), then, again, we have $\lambda_{DS}^{(X)} = -\lambda_{DL}^{(X)}$.

In difference with the CP -violating parameters (34), parameters (35) do not contain explicitly D -meson eigenstates. However, they use the states D and \overline{D} . Therefore, as explained above, they also cannot be determined unambiguously without some CP -prescription for D -meson eigenstates.

Such property is not unique for decays into neutral kaons. Consider, e.g., decays

$$D^0(\overline{D}^0) \rightarrow F \quad (36)$$

with amplitudes $a_D^{(F)}$, $a_{\overline{D}}^{(F)}$; here F is some final state with a definite CP -parity¹. The time distributions of decays (36) contain terms proportional to

$$\begin{aligned} & \text{Re} \lambda_D^{(F)} \sinh[(\Gamma_+ - \Gamma_-)t] \\ & \text{and} \\ & \text{Im} \lambda_D^{(F)} \sin[(m_+ - m_-)t], \end{aligned} \quad (37)$$

where

$$\lambda_D^{(F)} = \frac{q_D}{p_D} \frac{a_{\overline{D}}^{(F)}}{a_D^{(F)}}. \quad (38)$$

Expressions (37) show that the sign of $\text{Re} \lambda$ can be determined experimentally only if we know relation between eigenwidths and (approximate) CP -parities of eigenstates, while the sign of $\text{Im} \lambda$ needs relation between eigenmasses and CP -parities. An essential point is that in practice we cannot find these relations in decays (36) themselves (especially for masses), while cascade decays (1), (8) with intermediate neutral kaons may provide such possibilities.

5 Mass eigenstates with CP -violation

In preceding sections we have demonstrated that identification of D -eigenstates by their CP -parities, exact or approximate, requires two procedures for complete description of the states. One of them uses monotone (in t_D and t_K) terms of decay distributions and relates eigenwidths to the corresponding CP -parity eigenstates. Another procedure, by means of double oscillations (again, in t_D and

¹ Really we briefly repeat here a similar discussion of[3] for B -mesons.

t_K), relates eigenmasses to the eigenstates and determines which of them is heavier or lighter.

However, we have mentioned above that a rather familiar approach in the current literature is to identify eigenstates of heavy flavoured neutral mesons from the beginning by their masses, as heavier (e.g., D_h) or lighter (e.g., D_l) states. In such notations the kaon nomenclature would look as K_h, K_l instead of K_L, K_S correspondingly. Evidently, this approach is meaningful only if Δm can be measured, even if $\Delta\Gamma$ is too small for measuring. In this section we will consider D -meson eigenstates as identified by their masses.

Formally, cascade amplitudes and double-time distributions with such identification of eigenstates can be easily obtained from eqs. (9), (10) and (12)-(15), respectively. Of course, meaning of the indices \pm should be defined differently than in the preceding section. If we take, by definition, $\Delta m_D > 0$, then according to (16) we need to identify the states as

$$D_- \equiv D_h, \quad D_+ \equiv D_l \quad (39)$$

(similar to kaons). Therefore, we should rewrite eigenstate amplitudes as

$$\begin{aligned} a_{-L}^{(X)} &\rightarrow a_{hL}^{(X)}, & a_{-S}^{(X)} &\rightarrow a_{hS}^{(X)}, \\ a_{+L}^{(X)} &\rightarrow a_{lL}^{(X)}, & a_{+S}^{(X)} &\rightarrow a_{lS}^{(X)}. \end{aligned} \quad (40)$$

Here the first subscripts correspond to initial D -meson eigenstates being heavier or lighter, while the second ones are for final kaon eigenstates, Long- or Shortliving.

At first sight, the above change being supplemented by the substitution

$$\Gamma_- \rightarrow \Gamma_h, \quad \Gamma_+ \rightarrow \Gamma_l$$

should be quite sufficient. The situation, however, is not so simple. The real problem, as before, is how to construct procedures that determine, which of phenomenological amplitudes is which, and allow to relate physical eigenwidths and CP -properties with the mass labels of eigenstates.

To discuss this problem we, again, begin with a hypothetical assumption that every particular contribution in double-time distributions (12)-(15) can be separated experimentally by detailed study of double-time distributions for initial D - and \bar{D} -states. This assumes also that both Δm_D and $\Delta\Gamma_D$ are large enough to be measurable.

First of all, we separate terms having monotone behaviour in t_K (see eqs.(12), (14)). They correspond to D -meson decays with production of K_S or K_L , without their interference. Both cases lead to t_D -dependence of the form

$$\begin{aligned} &|a_1|^2 \exp(-\Gamma_1 t_D) + |a_2|^2 \exp(-\Gamma_2 t_D) \\ &+ 2|a_1 a_2| \cos(\phi_{12} + \Delta m_D t_D) \exp[-(\Gamma_1 + \Gamma_2) t_D / 2]. \end{aligned} \quad (41)$$

Here a_1 and a_2 are two amplitudes (e.g., $a_{hL}^{(X)}, a_{lL}^{(X)}$) for terms with the factor $\exp(-\Gamma_L t_K)$ in eqs.(12), (14)), ϕ_{12} is their relative phase; Γ_1, Γ_2 stay for Γ_h and/or Γ_l .

We see that the distribution (41), even having been ideally measured, would allow to determine the two amplitudes a_1 and a_2 (up to complex conjugation, because of possible change $a_1 \rightarrow a_2^*, \Gamma_1 \rightarrow \Gamma_2$) and relate them with eigenwidths Γ_1, Γ_2 , but could not show which of them is for heavier or lighter D -meson eigenstates. A formal reason is that the contributions (12) and (14) have symmetry properties: they do not change under substitution, e.g., $a_{+L}^{(X)} \rightarrow a_{-L}^{(X)*}, \Gamma_+ \rightarrow \Gamma_-$ (and/or similar for subscript S), with Δm_D staying unchanged. Thus, interchange of states D_+ and D_- (i.e. of states D_l and D_h) without changing Δm_D could not have any influence on contributions (12) and (14).

So, the t_K -monotone contributions in decay time distributions cannot discriminate the two eigenstates, heavier and lighter: they cannot relate the measured widths Γ_1 and Γ_2 to heavier or lighter eigenstates and cannot relate amplitudes to the corresponding eigenstate transitions. Of course, study of such terms would not be, nevertheless, useless: it can determine absolute values of all four amplitudes and two of their relative phases (one may note, however, that the phases can be determined in this way only up to signs).

Contributions (13), oscillating in t_K without oscillations in t_D , determine more of relative phases, but cannot yet, by themselves, distinguish amplitudes for initial states D_h or D_l . The reason is that these contributions also satisfy a symmetry property preventing, again, discrimination of D_{\pm} (i.e. of D_h, D_l): they do not change under substitution $a_{+L}^{(X)} \rightarrow a_{-L}^{(X)}, a_{+S}^{(X)} \rightarrow a_{-S}^{(X)}, \Gamma_+ \rightarrow \Gamma_-$. This symmetry is different from one discussed above. It works, nevertheless, for contributions (12) as well, but not for contributions (14). Therefore, contributions (13) together with (12) and (14) could, in principle, distinguish the states D_h and D_l and relate them to amplitudes and eigenwidths. Note, however, that the contributions (13), (14) vanish in the case of CP -conservation and, thus, are expected to be small (this brief discussion may be directly compared with a similar discussion in the preceding section).

The situation can be really resolved by contributions (15), oscillating in both t_K and t_D . These contributions by themselves violate the both above symmetries. Here the interchange of amplitudes and widths for h - and l -states would require simultaneous change of the relative sign between the known terms proportional to Δm_D and Δm_K . Therefore, here at last we can determine which two of four amplitudes correspond to decays of, say, D_h . After that, all the relative phases of the four amplitudes become unambiguous as well. Eigenwidths have been earlier related to definite amplitudes which now become specified as h - and l -amplitudes. So, due to contributions (15) the widths can be definitely related to h - and l -states. The CP -parity (exact or approximate) of eigenstates becomes also determined just by contributions (15), through relation between $|a_{lL}^{(X)} a_{hS}^{(X)}|$ and $|a_{lS}^{(X)} a_{hL}^{(X)}|$ which should correspond to inequality (32) (without substitution (40)).

When discussing CP -violation in terms of D_h, D_l the parameters (34) are not convenient, since without any spe-

cial model one does not know *a priori* which of amplitudes, e.g. $a_{hS}^{(X)}$ or $a_{lS}^{(X)}$, is larger in absolute value. In contrast, the parameters (35) are quite appropriate. One should note, however, that relations (17)-(22) between flavour and eigenstate amplitudes were written for the identification of D -eigenstates by their CP -parity (at least, in one pair of transitions $D_{\pm} \rightarrow K_S$ or $D_{\pm} \rightarrow K_L$). The substitutions (39), (40) should not be applied to them directly, before the amplitudes having been identified as described above in this section. Therefore, without using double-flavour oscillations one cannot unambiguously extract flavour-transition amplitudes from distributions (12)-(15) and, as a result, cannot unambiguously determine parameters (35) (and (38) as well). Similar discussions for B -mesons see in [3, 6–8].

We emphasize once more that there is no direct way to relate eigenwidths (even if measured) to a heavier (or lighter) eigenmass. The widths may be directly related only to amplitudes. The role of the coherent double-flavour oscillations is to relate the amplitudes (and, hence, the widths as well) to the mass eigenstates. The same note is true for CP -properties of the eigenstates. CP -violating parameters cannot be determined unambiguously without using the double-flavour oscillations.

Let us briefly discuss the problem of flavour amplitudes for transitions (2), (3) with mass labeling of D -meson eigenstates. Formally, they can be easily expressed through eigenstate amplitudes by eqs. (17)-(22). However, D -meson subscripts \pm in these relations correspond just to CP -properties of eigenstates and should not be changed according to substitution (39). Therefore, the flavour amplitudes can be determined unambiguously only when (approximate) CP -parities of the mass eigenstates have been measured, as can be done in double-flavour oscillations. The reason is the same as discussed in the preceding section: to construct correct flavour states (\overline{D} in the conventional approach) we need to know which of states, D_h or D_l , is (approximately) CP -even and/or CP -odd.

6 Concluding remarks. Strategy of measurements

In previous sections we have shown that coherent double-flavour oscillations suggest possibilities to solve various problems in D -meson physics. To understand which experiment may study this or that problem, we begin this section with estimating expected values of different effects.

The Minimal Standard Model leads to a natural estimate

$$\left| \frac{a_{DK}^{(X)}}{a_{\overline{DK}}^{(X)}} \right| \approx \left| \frac{a_{DK}^{(X)}}{a_{\overline{DK}}^{(X)}} \right| \sim \mathcal{O}(\tan^2 \theta_C),$$

with $\tan^2 \theta_C \approx 0.05$, where θ_C is the Cabibbo angle. This expectation corresponds to known experimental data [21] and, due to relations (17)-(22), leads to boundaries

$$|\phi_{SL}^{(Xf)}| \lesssim \mathcal{O}(\tan^2 \theta_C)$$

for the phase difference $\phi_{SL}^{(Xf)}$ in expression (29). The largest value of $\phi_{SL}^{(Xf)}$, admissible by this estimate, would be achieved at the relative phase of $a_{DK}^{(X)}$ and $a_{\overline{DK}}^{(X)}$ equal to $\pm\pi/2$. Considerations based on final state interactions tend to change this phase and reduce $\phi_{SL}^{(Xf)}$ even stronger.

For the quantity α_D , appearing in single-time distributions together with $\phi_{SL}^{(Xf)}$ (see expression (29)), the Minimal Standard Model gives very small expected values, typically $< 10^{-3}$. However, various hypotheses on New Physics may lead to larger values, up to $|\tan \alpha_D| = |x_D| \sim 0.1$. The present experimental data [21] still give a rather weak limitation

$$|\tan \alpha_D| < 0.09,$$

and cannot exclude such New Physics. So α_D could be of the same order as $\phi_{SL}^{(Xf)}$ or even higher, and the problem of their separation looks serious.

Distributions on the secondary-decay time t_K for decays (25) cannot, by themselves, separate $\phi_{SL}^{(Xf)}$ and α_D . This means that they cannot determine amplitudes $a_{DK}^{(X)}$ and $a_{\overline{DK}}^{(X)}$ and/or mass difference Δm_D (the same is true for using secondary decays of only K_S or K_L , or for total yields of decay products integrated over t_K). One could, however, try to interpret experimental results by applying additional hypotheses which should be checked. If, for instance,

$$|\alpha_D| \gg |\phi_{SL}^{(Xf)}|,$$

then the constant phaseshifts for oscillating terms in all decays (25) should be the same; $|\tan \alpha_D|$ should coincide with $|x_D|$ measured in semileptonic decays of neutral D -mesons (note that semileptonic decays are insensitive to the sign of x_D). If

$$|\alpha_D| \ll |\phi_{SL}^{(Xf)}|,$$

then useful would be comparison with similar phase shifts in the charged D -meson decays (30), having no mixing (these decays are of independent interest as well). Also useful could be studies of decays of neutral or charged D -mesons to charged kaons. They measure absolute values of amplitudes isotopically related to amplitudes of transitions (2), (3) for decays (25).

More accurate separation between different interference effects, mixing and/or suppressed vs. favoured transitions, can be achieved only by invoking information on double-time distributions. Of course, detailed studies of double-time oscillations require very high experimental statistics. One can imagine, however, that they would not be necessary. For example, comparison of t_K -distributions in various t_D -regions (say, $t_D \lesssim \tau_D$ and $t_D \gtrsim \tau_D$) could be sufficient at relatively moderate statistics. Treatment of the corresponding results could be simplified by taking into account the smallness of Δm_D and $\Delta \Gamma_D$. To achieve

more definite judgment on experimental availability of such studies one needs various Monte Carlo simulations. In any case, necessary CP -parities of heavier and lighter D -eigenstates (if they are not mode-dependent) could be measured in special experiments, and used afterwards in all other studies (just as done for kaons).

In summary, we have shown that the phenomenon of coherent double-flavour oscillations (CDFO) in cascade decays of heavy neutral flavoured mesons into intermediate neutral kaons is very useful to study heavy mesons and their decays. The phenomenon reveals itself mainly in double-time decay distributions (over the primary and secondary decay times). It gives, first of all, possibility to determine CP -parities (exact or approximate) of the heavy meson eigenstates, suggests new approaches to investigation of CP -violation and (especially for D -mesons) of suppressed flavour-transition amplitudes. It could also check consistency of various assumptions on the mesons.

On the other hand, CDFO appears to be inevitable to solve some problems unambiguously and in a model-independent way. They are, in particular, such important problems as the unambiguous measurement of CP -violating parameters and/or relation of the meson eigenwidths and eigenmasses. Another problem, specific for D -mesons, is study of doubly Cabibbo-suppressed transitions which are coherent with Cabibbo-favoured transitions in decays (of both neutral and charged D -mesons) to final states with neutral kaons. Such studies are very interesting by itself and may give evidence for New Physics, independent of (and additional to) CP -violation studies. Our main point is that extraction of both suppressed amplitudes and CP -violating parameters for neutral D -mesons appears impossible without investigation of CDFO. To separate effects of D -meson mixing and interference of suppressed vs. favoured amplitudes, such investigations for D -mesons, in contrast to B -mesons, require to know double-time decay distributions (i.e., over both primary and secondary decay times). For charged D -mesons one should also measure the secondary decay time distributions to achieve unambiguous extraction of suppressed amplitude in decays with neutral kaon production.

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